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1979

Online at <https://mpra.ub.uni-muenchen.de/24440/>

MPRA Paper No. 24440, posted 17 Aug 2010 12:42 UTC

# SIMULATION OF A NONLINEAR ECONOMETRIC MODEL

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This paper describes some analytic simulation experiments performed on a nonlinear macroeconomic model of the Italian economy. The proposed techniques extend to nonlinear models methods that are available, in the literature, for linear econometric models. The results can be profitably used either to validate the model or to evaluate the reliability of economic policy experiments.

## 1. INTRODUCTION

The analysis of the dynamic properties of a stochastic econometric model is an important source of information both for the validation of the model structure and for a more accurate insight into the problems related to the use of the model, such as in forecasting and in simulating alternative economic policies.

For linear models, these properties can be analytically derived; for example, analytical formulae are available for the direct computation of the following:

- reduced form variances;
- impact multipliers (which are a subset of the reduced form coefficients [5,p.508]) and related covariance matrix [8];
- variances of the forecast errors in one-step simulation [8], [1];
- interim multipliers and related covariance matrix [16];
- variances of the forecast errors in dynamic simulation [17];
- characteristics of the model in the frequency domain [5,ch.12].

In nonlinear models the same information cannot be obtained directly by analytical methods, and resort must be generally done to simulation techniques; stochastic simulation (Monte Carlo) has been deeply used for most of such purposes (see for example [1] for the computation of the reduced form variances and [9] for the computation of the power spectra), but also very promising seems to be the use of analytic simulation. Howrey and Klein, who proposed such a terminology [9], called "analytic simulation" the joint use of analytical formulae and numerical simulation techniques. They applied

this methodology to the derivation of the cyclical properties (evaluation of the power spectrum of the gross national product) of the nonlinear Wharton model. An analogous application was performed by the authors, in a joint paper [2], on the nonlinear model of the Italian economy developed by ISPE (Istituto di Studi per la Programmazione Economica) [14].

A similar algorithm, therefore still called analytic simulation, will be applied in this paper for different purposes; in all cases it is necessary to estimate the covariance matrix of the asymptotic distribution of nonlinear transformations of sample statistics. Let  $\bar{a}$  be a vector of statistics derived from a sample of length  $n$ , such that  $\sqrt{n}(\bar{a} - \text{plim } \bar{a})$  is asymptotically distributed as multivariate normal with zero means and covariance matrix  $\Sigma$ ; if  $v = g(\bar{a})$  is a continuous and differentiable transformation, then  $\sqrt{n}(g(\bar{a}) - g(\text{plim } \bar{a}))$  is asymptotically normally distributed with zero means and covariance matrix  $G\Sigma G'$ , where  $G$  is the matrix of the partial derivatives of the elements of  $v$  with respect to those of  $\bar{a}$  [19,p.383]. Numerical simulation can be used to compute these derivatives, for example as ratios of finite increments. Without going into details, it can be simply recalled that several statistics derived from a structural econometric model are continuous and differentiable transformations of the structural coefficients and that consistent and asymptotically normally distributed estimates can be derived for the coefficients (see, for example, the textbook by Dhrymes [5,pp.191, 216, 323 and 351]).

In this paper, analytic simulation will

be applied to the already mentioned model developed by ISPE, to compute:

- covariance matrix of the asymptotic distribution of the impact multipliers, in particular their asymptotic standard errors (section 3);
- covariance matrix of the asymptotic distribution of the dynamic interim multipliers, in particular their asymptotic standard errors (section 4);
- variances of the forecast errors in one-step simulation (section 5);
- variances of the forecast errors in dynamic simulation (section 6).

It must be pointed out that, before performing the computations on this model, the authors tested the proposed techniques on the linear Klein-I model, well known in the econometric literature, comparing the numerical results and the computational performances with those of the available analytical methods (proposed in [8], [16] and [17]). These experiments were extremely encouraging, since coincidence of results up to a minimum of 4-5 significant decimal digits was accompanied by a reduction of the computation time from a minimum of 20 times (for the standard errors of the impact multipliers) to a maximum of 400 times (for the standard errors of the dynamic multipliers). The improvement of performances is expected to increase even more for larger linear models (of course a comparison cannot be performed on this model, since it is nonlinear).

All the computations have been performed by means of a modified version of the program described in [3] on a computer IBM/370 model 168.

## 2. THE MAIN FEATURES OF THE MODEL

The nonlinear model used in this work is the annual model of the Italian economy developed by a team led by ISPE and described in [14]; it is linear in the parameters, nonlinear in the endogenous variables and dynamic for the presence of lagged endogenous variables.

Since the kind of analysis to be performed requires as starting point a consistent estimate of the structural coefficients, the ISPE model (originally estimated by OLS), has been re-estimated, for the period 1955-1976, by Two Stage Least Squares with Principal Components (2SLS-PC) according to the so called method 4 by Kloeck and Mennes [11]. The version of the model

used in the following experiments consists of 34 equations, 19 of which are stochastic; 45 are the exogenous variables and 75 are the estimated coefficients. The 75x75 asymptotic covariance matrix of the structural estimated coefficients has been computed by means of the formula proposed by Theil [19, p.500].

In the following sections, empirical results are displayed for some endogenous variables that, at least according to the purposes for which the model was built, can be regarded as possible targets in economic policy experiments [4]; in the same framework, the exogenous variables involved in the experiments can be considered the main economic policy instruments, whose effects and reliability are verified through the model.

The endogenous and exogenous variables considered in this paper are hereafter displayed and, in order to appreciate their magnitude, for the year 1977 the historical value is reported.

### ENDOGENOUS VARIABLES

CPNCF	- Private Consumption Net of Indirect Taxes (billions of 1970 liras), 1977=36647.
DXML	- Price Deflator for Exports of Manufactured Goods (1970=1), 1977=2.9285
IFIT	- Private Non-Residential Fixed Investment in Industry and Private Service Sector (billions of 1970 liras), 1977=6807.
LI	- Industrial Employment (thousands), 1977=7544.
MT	- Imports of Goods and Services (billions of 1970 liras), 1977=13806.
PCL	- Price Deflator for Private Consumption Cross of Indirect Taxes (1970=1), 1977=3.0154
VAP	- Gross Output in the Private Sector (billions of 1970 liras), 1977=55818.
XT	- Exports of Goods and Services (billions of 1970 liras), 1977=16971.

### EXOGENOUS VARIABLES

ACSIM	- Social Security Contribution Rate in Manufacturing Industry, 1977=.44375
APA	- Intermediate Consumption in the Public Sector (billions of 1970 liras), 1977=2053.
ATI	- Direct Taxes Rate, 1977=.18236
ERL\$	- Exchange Rate USA Dollar/Lira, 1977=882.26

- IES - Fixed Investment in Agriculture and Public Sectors (billions of 1970 liras), 1977=1939.  
 LPA - Employment in the Public Sector (thousands), 1977=2375.6  
 TRI - Subsidies to Production (billions of current liras), 1977=4595.

### 3. IMPACT MULTIPLIERS AND ASYMPTOTIC STANDARD ERRORS

Multipliers analysis is frequently performed either when validating an econometric model or when using the model for economic policy experiments. In both cases additional information concerning the standard error of each multiplier could be interesting. In fact, economic policy experiments should not be based on policy instruments with multipliers not significantly different from zero (for reasonable significance levels); at the same time, in the validation phase, a model could not be accepted if multipliers with the wrong sign (wrong with respect to the underlying economic theory) are significantly different from zero.

Attempts of deriving the small-sample distribution of the impact multipliers have been performed in the literature (see, for example, [15]) using Monte Carlo methods. These methods, however, have a major drawback in the possible non-existence of finite moments in the small-sample distribution of the structural and reduced form coefficients, even for linear models when estimated with simultaneous consistent methods [12], [13]. A truncation must be, therefore, performed in the tails of the pseudo-random disturbances to be used in the Monte Carlo experiment [18, p.1004], thus involving some arbitrariness.

As suggested by Theil [19, p.377], resort to asymptotic distribution could be preferable. The problem of computing the asymptotic standard errors of impact multipliers was solved in 1961 by Goldberger, Nagar and Odch [8] for linear models; the proposed formulae, in order to be applicable to nonlinear models, would require an explicit linearization of the model, thus making extremely laborious the process even for small models.

Analytic simulation overcomes most of the difficulties allowing a fast and reliable computation for even moderately complex models. Without going into details, the method used in this context

is similar to the one described in [1]. It is based on the numerical computation of partial derivatives of the impact multipliers with respect to the structural coefficients, using the differences between a control solution and a set of disturbed solutions.

The tables from 1 to 7 display some impact multipliers and estimated asymptotic standard errors for the year 1977 (which is the first year outside the sample period used in the coefficient estimates).

Table 1

Impact Multipliers and Asymptotic Standard Errors of the Variable ACSIM

Variab.	Multip.	Std.Err.
CPNCF	-1054.64	3306.1
DXML	1.18256	0.3763
IFIT	-720.208	375.65
LI	-325.595	230.15
MT	-2390.05	1748.0
PCL	0.34705	0.2877
VAP	-4712.25	2969.0
XT	-4584.81	2053.4

Table 2

Impact Multipliers and Asymptotic Standard Errors of the Variable APA

Variab.	Multip.	Std.Err.
CPNCF	0.36635	0.1416
DXML	-0.000064	.00002
IFIT	0.12782	0.0320
LI	0.08524	0.0296
MT	0.54015	0.0869
PCL	-0.000041	.00001
VAP	1.19713	0.1745
XT	0.23695	0.1069

Table 3

Impact Multipliers and Asymptotic Standard Errors of the Variable ATI

Variab.	Multip.	Std.Err.
CPNCF	-40139.2	14394.
DXML	3.18613	1.1273
IFIT	-3785.19	1473.7
LI	-2270.36	1131.0
MT	-20442.1	7175.0
PCL	5.77375	0.9983
VAP	-34473.5	12361.
XT	-12352.6	5914.7

Table 4

Impact Multipliers and Asymptotic  
Standard Errors of the Variable  
ERL\$

Variab.	Multip.	Std.Err.
CPNCF	-2.96307	1.1044
DXML	0.00210	.00040
IFIT	-0.19979	0.2583
LI	0.06702	0.1239
MT	0.06082	1.0376
PCL	.000115	0.0001
VAP	0.78563	1.7279
XT	4.61899	2.1621

Table 5

Impact Multipliers and Asymptotic  
Standard Errors of the Variable  
IES

Variab.	Multip.	Std.Err.
CPNCF	0.32972	0.1264
DXML	-.000049	.00002
IFIT	0.10048	0.0258
LI	0.06669	0.0236
MT	0.60945	0.0748
PCL	-.000037	.00001
VAP	1.01657	0.1478
XT	0.19290	0.0877

Table 6

Impact Multipliers and Asymptotic  
Standard Errors of the Variable  
LPA

Variab.	Multip.	Std.Err.
CPNCF	1.42242	0.5011
DXML	-.000046	.00002
IFIT	0.10445	0.0452
LI	0.07031	0.0351
MT	0.64370	0.2366
PCL	-.000022	.00003
VAP	1.07217	0.3973
XT	0.18003	0.1115

Table 7

Impact Multipliers and Asymptotic  
Standard Errors of the Variable  
TRI

Variab.	Multip.	Std.Err.
CPNCF	0.35990	0.1302
DXML	-.000029	.00001
IFIT	0.03393	0.0133
LI	0.02035	0.0101
MT	0.18329	0.0650
PCL	-.000052	.00001
VAP	0.30910	0.1117
XT	0.11075	0.0533

Several interesting considerations can be derived from the results in tables 1-7. Some of them are, for example, the following.

- Total imports (MT) seem to be not significantly influenced by the exchange rate (ERL\$); the wrong sign in the multiplier is not enough to reject the model, since the multiplier is not significantly different from zero. On the other side, total exports (XT) seem to be significantly influenced by the exchange rate.
- The price effect (due to foreign trade substitution) on the private domestic production (VAP) seems to be not reliable; in fact the multiplier of ERL\$, for VAP, is not significantly different from zero.
- For supporting investment (IFIT), employment (LI) and production (VAP), APA, IES, LPA, TRI and ATI seem to be much more reliable instruments than ACSIM.

#### 4. INTERIM MULTIPLIERS AND ASYMPTOTIC STANDARD ERRORS

A complete multipliers analysis should take into account, besides the impact effects, also the effects of changes in the exogenous variables on the endogenous in subsequent periods. Goldberger defines delay or interim multiplier, at lag  $k$ , the change in (the expected value of) an endogenous variable at time  $t$  due to a unit change in an exogenous at time  $t-k$  [6], [7].

It is frequent the case of multipliers which change in value and sign when passing from lag zero (impact) to a lag of several periods. An economic policy which takes into account only the values of the impact multipliers could even lead to perverse results if, after few periods, the interim multipliers change of sign, provided this change is significant. The computation of the interim multipliers, therefore, should be accompanied by an estimate of their standard errors.

Once again it is possible to overcome most of the problems connected to finite sample distribution by computing the asymptotic standard errors; and once again this can be based on the numerical computation of the partial derivatives of the multipliers with respect to the structural coefficients. This method allows to extend to nonlinear models the procedure proposed by Schmidt [16] for linear models and allows a considerable simplification in the computation.

Table 8

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
ACSIM from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	-915.780	3166.2
	76	-2590.53	2492.6
	77	-1460.36	1004.1
DXML	75	0.78895	0.2507
	76	0.12895	0.1086
	77	-0.14765	0.1484
IFIT	75	-769.748	407.38
	76	-1869.39	1069.6
	77	-1458.17	738.80
LI	75	-337.364	241.71
	76	-670.120	388.19
	77	-335.209	198.83
MT	75	-2131.44	1633.1
	76	-3963.97	2410.6
	77	-2079.07	1175.7
PCL	75	0.22697	0.1868
	76	0.24849	0.1398
	77	0.08001	0.0882
VAP	75	-4450.15	2897.1
	76	-5612.66	2899.1
	77	-803.116	1110.3
XT	75	-4158.33	1925.7
	76	-4765.20	2396.6
	77	47.0815	811.34

Table 10

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
ATI from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	-38299.6	13759.
	76	-34603.7	8283.8
	77	-10735.1	6787.9
DXML	75	2.14314	0.7666
	76	0.67720	0.7129
	77	-0.80617	0.7004
IFIT	75	-3985.93	1550.3
	76	-12235.4	3919.7
	77	-7651.96	3390.5
LI	75	-2424.81	1179.1
	76	-4096.13	1355.8
	77	-1555.25	1103.5
MT	75	-18917.5	6729.7
	76	-28309.8	6080.6
	77	-13436.7	6683.9
PCL	75	3.77205	0.6735
	76	2.05715	0.6225
	77	0.76489	0.7212
VAP	75	-33451.2	12046.
	76	-31991.9	9598.1
	77	-3545.65	8141.8
XT	75	-11295.9	5559.3
	76	-14317.2	7603.5
	77	379.719	5373.1

Table 9

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
APA from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	0.39295	0.1508
	76	0.35470	0.1191
	77	0.00457	0.1266
DXML	75	-0.000004	.00001
	76	0.000001	.00002
	77	0.000003	.00002
IFIT	75	0.14078	0.0355
	76	0.39985	0.0894
	77	0.09215	0.0596
LI	75	0.09601	0.0322
	76	0.12007	0.0427
	77	0.00865	0.0199
MT	75	0.54358	0.0901
	76	0.44237	0.1159
	77	0.06681	0.1350
PCL	75	-0.00003	.000008
	76	-0.00002	.000009
	77	.000009	.00001
VAP	75	1.24080	0.1846
	76	0.49972	0.1775
	77	-0.18553	0.1834
XT	75	0.24244	0.1120
	76	0.19832	0.1331
	77	-0.19659	0.1607

Table 11

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
ERL\$ from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	-3.51076	1.3526
	76	-0.38977	2.0730
	77	0.52063	0.8247
DXML	75	0.00191	0.0003
	76	-0.00011	0.0001
	77	0.00014	0.0001
IFIT	75	-0.35582	0.4024
	76	0.74720	0.8794
	77	1.11390	0.8328
LI	75	0.09023	0.1756
	76	0.37565	0.3609
	77	0.25018	0.2073
MT	75	0.06209	1.3018
	76	2.19453	2.3860
	77	1.28866	1.1769
PCL	75	0.00014	0.0001
	76	-0.00003	0.0001
	77	.000008	.00008
VAP	75	0.98330	2.2604
	76	4.23068	3.3005
	77	0.12802	0.6808
XT	75	5.73223	2.7716
	76	6.31062	3.3190
	77	-0.08499	0.5393

Table 12

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
IES from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	0.34536	0.1318
	76	0.25012	0.1100
	77	-0.03516	0.0921
DXML	75	-0.00003	0.0000
	76	0.00001	0.0000
	77	0.00002	0.0000
IFIT	75	0.11251	0.0289
	76	0.30019	0.0672
	77	0.00746	0.0410
LI	75	0.07636	0.0259
	76	0.07777	0.0358
	77	-0.01432	0.0154
MT	75	0.60480	0.0769
	76	0.43735	0.0909
	77	-0.00543	0.0992
PCL	75	-0.00002	0.0000
	76	-0.00001	0.0000
	77	0.00000	0.0000
VAP	75	1.05715	0.1554
	76	0.22956	0.1343
	77	-0.21375	0.1382
XT	75	0.19933	0.0924
	76	0.12928	0.0966
	77	-0.17724	0.1239

Table 14

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
TRI from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	0.51371	0.1847
	76	0.46413	0.1107
	77	0.14398	0.0909
DXML	75	-0.00003	.00001
	76	-0.00001	.00001
	77	0.00001	.00001
IFIT	75	0.05346	0.0208
	76	0.16411	0.0526
	77	0.10263	0.0454
LI	75	0.03252	0.0158
	76	0.05494	0.0181
	77	0.02085	0.0147
MT	75	0.25374	0.0903
	76	0.37972	0.0816
	77	0.18022	0.0896
PCL	75	-0.00005	.00001
	76	-0.00003	.00001
	77	-0.00001	.00001
VAP	75	0.44868	0.1617
	76	0.42910	0.1286
	77	0.04754	0.1091
XT	75	0.15151	0.0746
	76	0.19203	0.1021
	77	-0.00509	0.0720

Table 13

Interim Multipliers and Asymptotic  
Standard Errors of the Variable  
LPA from 1975 to 1977

Variab.	Year	Multip.	Std.Err.
CPNCF	75	1.47314	0.5197
	76	0.61005	0.5060
	77	-0.66832	0.3911
DXML	75	-0.00003	.00002
	76	0.00006	.00004
	77	0.00013	.00006
IFIT	75	0.11986	0.0517
	76	0.31012	0.1078
	77	-0.10097	0.1576
LI	75	0.08254	0.0402
	76	0.08579	0.0400
	77	-0.07697	0.0661
MT	75	0.65500	0.2410
	76	0.51933	0.1636
	77	-0.53086	0.4238
PCL	75	-0.00001	.00002
	76	0.00001	.00002
	77	0.00012	.00006
VAP	75	1.14325	0.4233
	76	0.28740	0.3965
	77	-1.18213	0.3917
XT	75	0.19431	0.1187
	76	-0.07810	0.1447
	77	-0.89757	0.2427

The tables from 8 to 14 display some of the interim multipliers and their asymptotic standard errors for the years 1975, 1976 and 1977. All these multipliers are related to a change of the exogenous variables in the year 1975.

Several interesting considerations can be derived from the results in the tables 8-14.

- For some variables there are changes (after one or two periods) in the sign of the multipliers, but in general these multipliers are not significantly different from zero; on the other hand, for the case in which the change is significantly different from zero (i.e. the multiplier of LPA for the variable DXML in 1977), some doubts arise on the correctness of the sign of the multiplier in 1975.
- The exchange rate (ERL\$), that has significant impact effects for some variables only, for the same variables loses any reliability after one period; in particular this occurs also for DXML, that is a variable to which ERL\$ should be strongly and significantly connected.
- The instruments APA, ATI, IES, TRI, in general used for supporting investment (IFIT), employment (LI) and production (VAP), maintain the reliability of

their effects after one period (see their multipliers and related standard errors in 1976), but apart some cases (see multipliers of ATI and TRI for the variable IFIT), they lose their reliability after two periods (in 1977); furthermore, there is sometimes a change in the sign of the effects, but in these cases the multipliers are not significantly different from zero.

##### 5. VARIANCES OF THE FORECAST ERRORS IN ONE-STEP SIMULATION

The assumption of independence between structural disturbances in different periods allows to decompose the forecast errors (difference between the values of each endogenous variable computed with the model and observed in the forecast period) as the sum of two independent vectors of random variables; these two components are, respectively, due to errors in the estimated coefficients and to the structural disturbances.

The computation of the variances of the two components of the forecast errors in the one-step simulation is a typical application of analytic simulation. For nonlinear models the method has been recently proposed by the authors together with an application to the Klein-Goldberger model [1].

For the component due to error in the coefficient estimates, the procedure requires: 1) the knowledge of the complete asymptotic covariance matrix of the structural coefficients  $\hat{\beta}$  (directly supplied by system estimation methods such as Three Stage Least Squares or Full Information Maximum Likelihood, or properly computed, as in the case of this model where, as already mentioned, the formula in [19, p.500] for Two Stage Least Squares has been used); 2) the computation of the matrix  $(G)$  of the partial derivatives in the solution point of the endogenous variables with respect to the structural coefficients. The quadratic forms which are the diagonal elements of the matrix  $G\hat{\Sigma}G'/T$  ( $T$  being the sample length) are the asymptotic variances of the component of the forecast errors due to errors in the coefficient estimates [1].

For the component of the forecast errors due to the structural disturbances the procedure requires: 1) the knowledge of the covariance matrix of the structural disturbances; 2) the computation of the partial derivatives of the endogenous variables with respect to the structural disturbances. The corresponding

quadratic forms are, this time, approximated values of the variances of this component of the forecast errors; the approximation, for several models tested by the authors, including the ISPE model, is better than the approximation obtained with several thousands replications of stochastic simulation and the computation is considerably faster (see, for example, [1]).

For some of the endogenous variables of the ISPE model, table 15 displays the variances of the two components of the forecast errors in the one-step simulation at 1977. The square roots of the sum of the variances of the two components are displayed on the right hand side of table 16 together with the observed and the computed values of the variables.

Table 15

Forecast Errors in 1977  
One-Step Simulation

Variab.	Var.due to Coeff.	Var.due to Disturb.
CPNCF	148997.	281302.
DXML	0.00609	0.00551
IFIT	68828.9	76038.0
LI	5175.05	20184.1
MT	114621.	164885.
PCL	0.00339	0.00249
VAP	283380.	370629.
XT	313773.	358461.

Table 16

Forecast Errors in 1977  
One-Step Simulation

Variab.	Observed Value	Computed Value	Std.Err.of Forec.Err.
CPNCF	36647.0	3676924	655.9
DXML	2.92850	2.90467	0.107
IFIT	6807.00	7134.78	380.6
LI	7544.00	7706.78	159.2
MT	13806.0	14298.6	528.6
PCL	3.01539	3.03318	0.076
VAP	55818.0	55372.0	808.7
XT	16971.0	16676.5	819.8

##### 6. VARIANCES OF THE FORECAST ERRORS IN DYNAMIC SIMULATION

As in the case of section 5, also in a multiperiod dynamic simulation the forecast errors can be decomposed into two terms due, respectively, to error in the coefficient estimates and to the structural disturbances and, under the



same assumptions, the two terms are independent when forecasting outside the sample period used in the estimation phase.

The computation of the variances of the component due to error in the coefficients is quite similar to the case of one-step simulation; the only difference is that the control and disturbed solutions, used to compute the partial derivatives of the endogenous variables with respect to the structural coefficients, must be dynamic instead of one-step.

Also for the variances of the second component the computation is similar to the case of one-step simulation. It must, however, be pointed out that the behaviour of nonlinear models is not symmetric, so that the derivatives of the endogenous variables at time  $t$  with respect to the structural disturbances at time  $t-k$  are generally different from those of the variables at time  $t+k$  with respect to the disturbances at time  $t$ . Facing properly this problem, the approximation in the results of the ISPE model has been found by the authors better than the approximation obtained with several thousands replications of stochastic simulation.

The results in tables 17 and 18 are referred to the year 1977 after a 3 years dynamic simulation (starting from 1975). Therefore, there is an approximation in the results, since the years 1975 and 1976 belong to the sample period, so that the disturbances are not independent from the estimated coefficients, while they would be when forecasting outside the sample period.

It is interesting to observe from table 15 that, while the component due to the structural coefficients is generally greater than the other, it is smaller for the price variables DXML and PCL; this effect on DXML and PCL is probably due to inflation. In fact, as in the case of the single equation forecast error [10, p.261], the variance due to error in the coefficient estimates is quite small in the neighbourhood of the sample means of the explanatory variables, but it becomes larger and larger when the forecast is performed in correspondence of values that are far from the mean values; in this case, DXML and PCL are directly influenced by other price variables and by the cost of labour, which are variables with large inflation effect in the last years. In case of dynamic simulation (table 17), also for the price variables the

component of the variance due to the structural disturbances is always greater than the other.

The size of the standard errors in tables 16 and 18 gives a clear idea of the reliability of the forecasting produced by the model.

Table 17

Forecast Errors in 1977  
Dynamic Simulation from 1975

Variab.	Var.due to Coeffic.	Var.due to Disturb.
CPNCF	536135.	749355.
DXML	0.00889	0.01112
IFIT	119973.	136689.
LI	8898.94	27301.4
MT	465306.	526348.
PCL	0.01288	0.01383
VAP	632012.	978373.
XT	618290.	733583.

Table 18

Forecast Errors in 1977  
Dynamic Simulation from 1975

Variab.	Observed Value	Computed Value	Std.Err.of Forec.Err.
CPNCF	36647.0	37830.2	1134.
DXML	2.92850	2.79305	0.141
IFIT	6807.00	7309.72	506.6
LI	7544.00	7773.69	190.3
MT	13806.0	15100.2	995.8
PCL	3.01539	2.88658	0.163
VAP	55818.0	56501.5	1269.
XT	16971.0	17452.1	1163.

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